HW 10

① (a) Signal is proportional to "interference term":

$$S \propto 2\sqrt{I_1 I_2}\cos\delta$$

Noise is proportional to the sum of intensities:

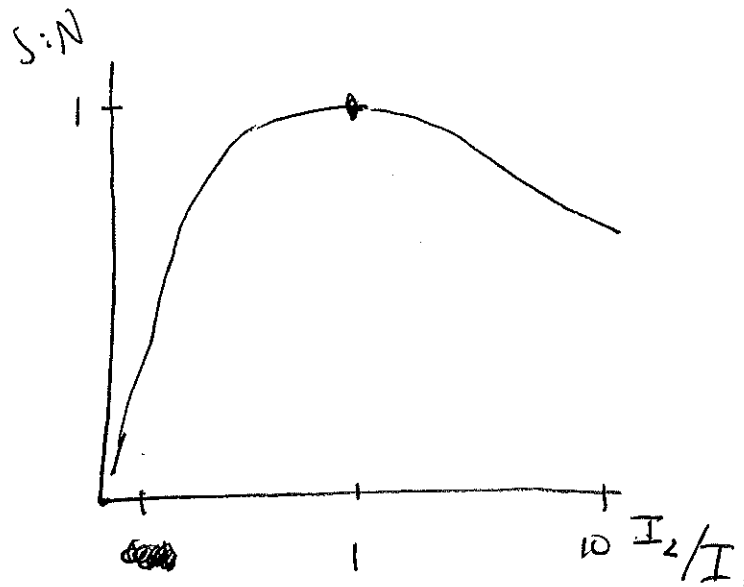
$$N \propto I_1 + I_2$$

→ Assume we are at complete constructive interference (i.e. $\cos\delta = 1$)

$$\boxed{\frac{\text{Signal}}{\text{Noise}} = S:N = \frac{2\sqrt{I_1 I_2}}{I_1 + I_2}}$$

(b)

I_2	S/N
0	0
$\frac{1}{2} I_1$	0.94
I_1	1
$2 I_1$	0.94
$10 I_1$	0.57



$\boxed{\text{Signal to noise is maximum for } I_2 = I_1.}$

(2) (a) $I = I_{central} \cos^2\left(\frac{y\pi}{\lambda}\frac{a}{L}\right)$ ← cosine squared pattern

zeroth bright fringe @ $y_0 = 0$

1st " " @ $y_1 = \frac{\lambda L}{a}$

2nd " " @ $y_2 = 2\frac{\lambda L}{a}$

$$\underline{y_1 - y_0} = \underline{y_2 - y_1} = \frac{\lambda L}{a} = \frac{(543\,nm)(1.00\,m)}{(0.500\,mm)} = \boxed{1.09\,mm}$$

(b) $I = I_{central}\left(\frac{\sin\beta}{\beta}\right)^2$ ← sinc squared pattern

where $\beta \approx \frac{y\pi}{\lambda}\cdot\frac{D}{L}$ for small θ

$Sinc^2$ function has maxima at

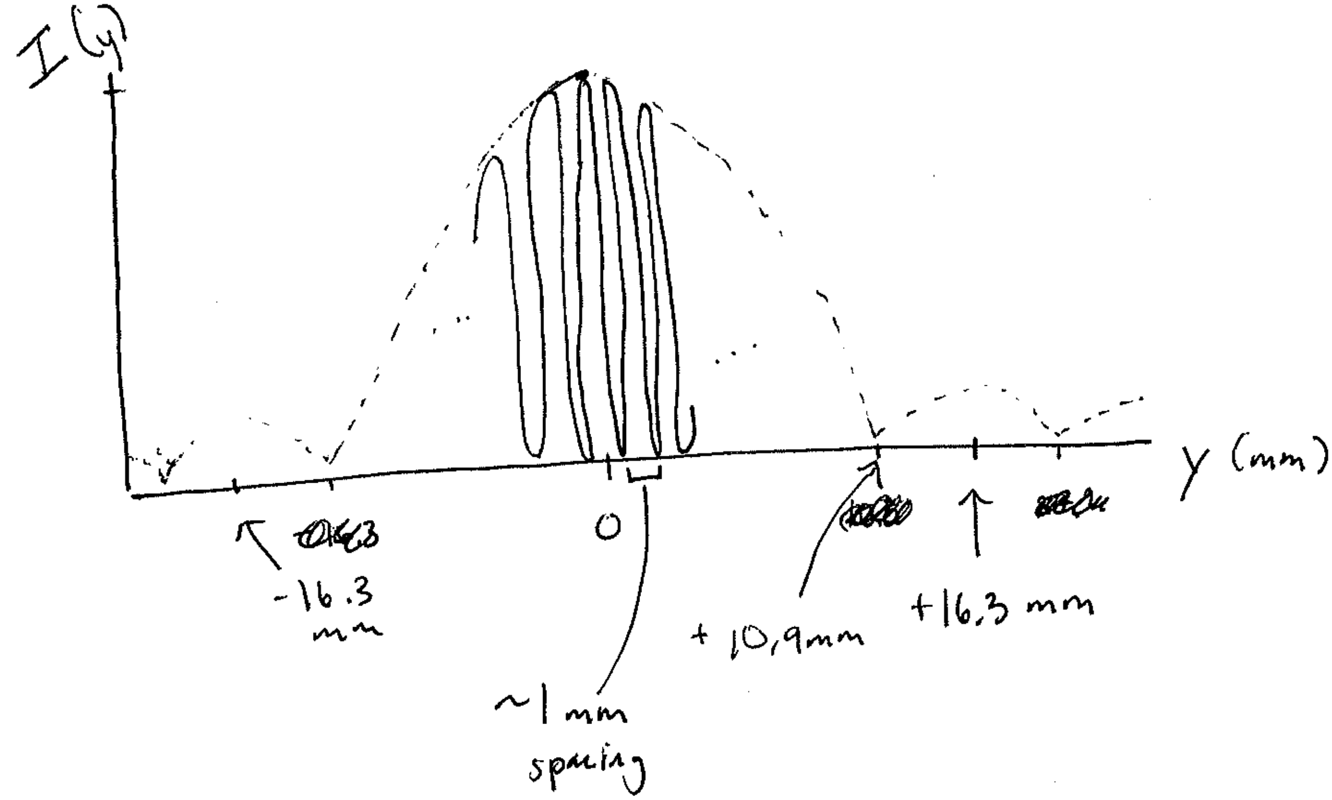
- $y_0 = 0$
- $y_1 = \frac{3}{2}\frac{\lambda L}{D}$ (when $\beta = \frac{3\pi}{2}$)
- $y_2 = \frac{5}{2}\frac{\lambda L}{D}$ (when $\beta = \frac{5\pi}{2}$)

$$y_1 - y_0 = \frac{3}{2}\cdot\frac{(543\,nm)(1.00\,m)}{(0.050\,mm)} = \boxed{16.3\,mm}$$

$$y_2 - y_1 = \left(\frac{5}{2} - \frac{3}{2}\right)\frac{\lambda\ L}{D} = \boxed{10.9\,mm}$$

(2 continued)

(c) The pattern will be $I \propto \text{sinc}^2 \beta \times \cos^2\left(\frac{y a \pi}{\lambda L}\right)$.



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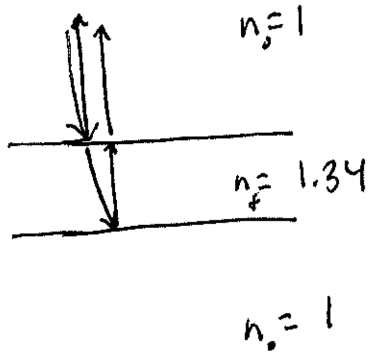
(3)



$$\Delta OPL = 2 t n_f + \frac{\lambda}{2} = m\lambda$$

$m \in$ integer $\rightarrow$ constructive IF

$$\Delta OPL = 2 t_{min} n_f + \frac{\lambda}{2} = \lambda$$

$$t_{min} = \frac{\lambda/2}{2 n_f} = \frac{\lambda}{4 n_f} = \boxed{118\ nm}$$

(4) (1) Compare the case of reflection off the substrate into air to reflection off the film into air.

$$r_{film \to air} \overset{??}{<} r_{subs. \to air}$$

$$\frac{n_f - 1}{n_f + 1} \overset{?}{<} \frac{n_s - 1}{n_f + 1}$$

$$(n_s + 1)(n_f - 1) < (n_s - 1)(n_f + 1)$$

$$n_s n_f + n_f - n_s - 1 < n_s n_f - n_f + n_s - 1$$

$$\boxed{n_f - n_s < n_s - n_f}$$ True for $1 < n_f < n_s$

(2) If the thickness of the film is $\frac{\lambda_f}{4}$ and $1 < n_f < n_s$, the reflection off of the film-substrate interface will always be exactly π out of phase with the film-air reflection. Thus, $r_{film \to air}$ would get <u>reduced</u>.

⑤ refractive index: $n_f = \sqrt{n_o n_g} = \boxed{1.24}$

thickness : $\Delta OPL = \underline{2 t n_f} = \frac{\lambda m}{2}$ ← destructive IF

$$t_{min} = \frac{\lambda}{4 n_f} = \boxed{109\,nm}$$

⑥ (a) $F = \left(\frac{2r}{1-r^2}\right)^2 = \left(\frac{2 \cdot 0.8944}{1-(0.8944)^2}\right)^2 = \boxed{79.96}$

(b) $\mathcal{F} = \frac{\pi}{2}\sqrt{F} = \boxed{14.05}$

(c) $FWHM = \frac{FSR}{\mathcal{F}} \approx \boxed{107\ MHz}$

(also, $\frac{4}{\sqrt{F}} \approx \boxed{0.447\ \text{radians}}$) (also $FWHM \approx \boxed{0.1076\ pm}$)

(d) $FSR = \frac{\text{speed of light}}{2\,(10\ cm)} = \boxed{1.50\,GHz}$

(also, $FSR \approx \boxed{6.28\ \text{radians}}$)

(also, $FSR \approx \boxed{1.51\ pm}$)

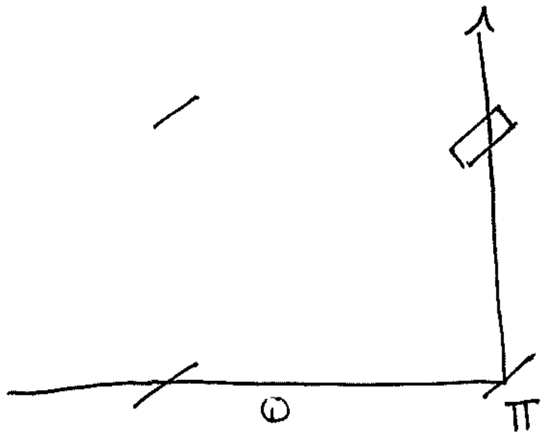
→ lots of choices for units!

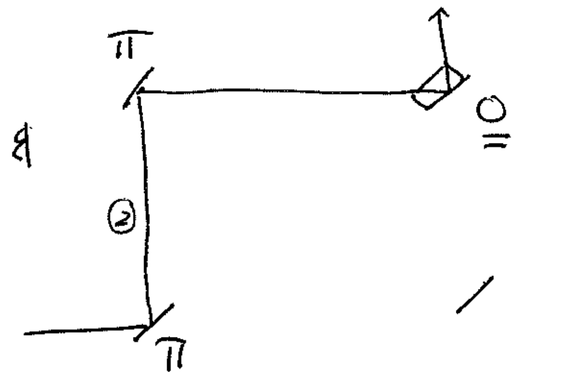
(e) $I_t = I_i \frac{1}{1 + F \sin^2 \frac{\delta}{2}}$

$$C = \frac{(I_t/I_i)_{max}}{(I_t/I_i)_{min}} \approx \boxed{81} \leftarrow 1+F$$

⑦ <u>Case 1</u>: reflect off <u>back</u> of last beamsplitter (as pictured)

(a) <u>PD1</u>:



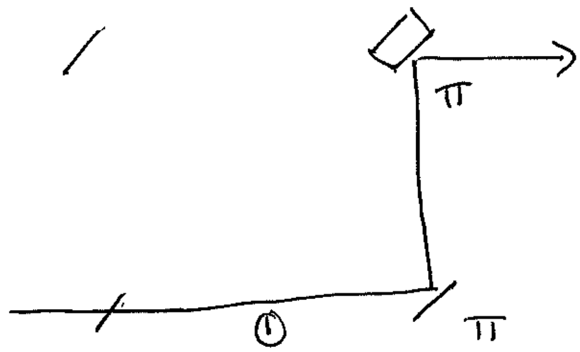


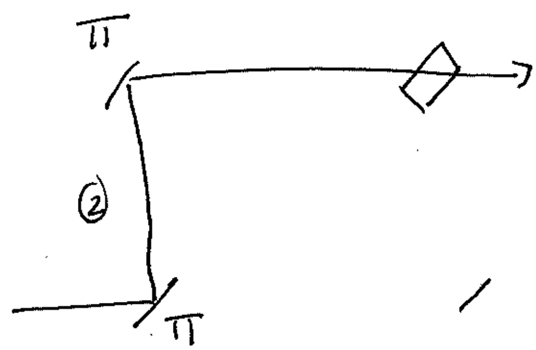
Beam 1 has π shift

Beam 2 has $2\pi = 0$ shift

→ PD1 sees <u>no</u> light

(b) <u>PD2</u>:





Beam 1 has $2\pi = 0$ phase shift

Beam 2 <u>also</u> has $2\pi = 0$ phase shift

→ PD2 see 100% of light

<u>Case 2</u>: if you are instead looking at the reflection off the beamsplitter <u>front</u>, PD1 sees 100% and PD2 sees 0%.

E.C. 1

- The parts of the beam that are not affected by the candle act same as in #7: bright at PD2 and dark at PD1. This is why image at PD1 has a <u>dark</u> background and the other has a <u>light</u> background.
- The candle reduces the air density of nearby air, causing a shift in index of refraction. This only happens in beam "1" (see #7) and it only happens near the flame.
- The shifting of n causes an interference "image" of the heat caused by the candle.
- The image at "PD1" is the <u>negative</u> of the image at "PD2". (Energy / Intensity is conserved.)

$\boxed{\text{E.C. 2}}$

(a) You can start with $Q = \frac{\nu_0}{FWHM}$. (from class)

Then $Q = \frac{\nu_0}{FSR} \frac{FSR}{FWHM} = \mathcal{F} \frac{\nu_0}{FSR}$

(b) $Q = \frac{545 \times 10^{12}\ Hz}{107 \times 10^{6}\ Hz} \approx 5$ million

(c) $\mathcal{F} = Q \frac{FSR}{\nu_0} = (1 \text{ million}) \frac{1.5\ GHz}{545\ THz} = 2.75$

$\mathcal{F} = \frac{\pi}{2}\sqrt{F} = \frac{\pi}{2} \frac{2\sqrt{R}}{1-R} = 2.75$

$R \approx 0.33$ or 33% will work for such a long cavity